Arithmetic operations

Objectives

At the end of the lecture students may learn:

1- Arithmetic operation on binary numbers system.

2- Arithmetic operation on octal numbers system.

3- Arithmetic operation on hexadecimal numbers system.

4- Arithmetic operation on decimal numbers system.

5- Examples.

1- Arithmetic operation on binary numbers system

The addition operation

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 0$ with carry = 1

Example-1: $(111011)_2 + (10010)_2 = (\qquad)_2$

solution:

$$
\begin{array}{r}
1\ 1\ 1\ 0\ 1\ 1 \\
+ \quad 1\ 0\ 0\ 1\ 0 \\
\hline
1\ 0\ \ 0\ 1\ 1\ 0\ 1
\end{array}
$$

Note that the carry = 1

Example-2: $(111.1101)_2$ + $(11.01)_2$ = ($)_2$

$$
\begin{array}{rr}
 & 111.1101 \\
+ & 11.0100 \\
\hline
 & 1011.0001
\end{array}
$$

The result is $(1011.0001)_2$

Note that the carry=1

Example-3: $(10111110)_2$ + $(10001101\ )_2$ =($\quad$ $)_2$

$$
\begin{array}{r}
1\,0\,1\,1\,1\,1\,1\,0 \\
+ \quad 1\,0\,0\,0\,1\,1\,0\,1 \\
\hline
1\,0\,1\,0\,0\,1\,0\,1\,1
\end{array}
$$

The result $(101001011)_2$

Note that the carry=1

The subtraction operation

0 - 0 = 0

1 - 0 = 1

0 - 1 = 1..

(with brow 1)

1 - 1 = 0

Example-4

$(10110)_2$ - $(1001)_2$ =($)_2$

```
   1 0 1 1 0
-    1 0 0 1
------------
   0 1 1 0 1
```

The result $(01101)_2$

Note that the brow=0

Subtraction with complements

The direct method of subtraction taught in elementary schools uses the borrow concept. This method, works well when people perform subtraction with paper and pencil. However, when subtraction is implemented with digital hardware , the method is less efficient than the method uses complements.

A binary No-s complement

- One' (1's) complement (0 → 1, 1 → 0)
- Two's (2's) complement (1's complement +1)

Example-5: Find 1's complement of the binary number (1110101)

Solution: 1110101

1's complement 0001010

Example-6: Find $(11011)_2$ $-(1101)_2$ by using 1's complement

Solution:

11011- 01101= 11011 + 10010= 01101 with carry=1

Note: If carry=1 then the result is equal to(addition result + carry) and the result is positive.

$$\begin{array}{r} 01101 \\ +\quad\quad 1 \\ \hline 01110 \end{array}$$

Example-7: Find $(1011)_2 - (1110)_2$ by using 1's complement

Solution: 1011-1110= 1011+ 0001=1100 with carry=0

Note: If carry=0 then the result is equal to 1's complement of the addition result and the result is negative.

The addition result 1100

1's complement   0011

The result = - 0011

Example-8: Find the 2's complement to number $(110001)_2$

Solution:

$$
\begin{array}{lr}
 & 110001 \\
\text{1's complement} & 001110 \\
+ & 1 \\
\hline
 & 001111
\end{array}
$$

The 2's complement of $(110001)_2$ is $(001111)_2$

Example-9: Find $(11100)_2$ - $(0111)_2$ by using 2's complement

Solution:

11100-00111=11100+11001 =10101withcarry=1

Note: If carry=1 then the result is equal to addition result and the result is positive

Example-10: Find $(1111)_2$ –(10000) by using 2's complement

Solution: 1111-10000=1111+10000=11111 with carry=0

If carry=0 then the result is equal to 2's complement of the edition result and the result is negative

	11111
1's complement	00000
+	1
	00001 and negative

Example-11: Find (01001-1000) by using 2's complement

9	01001
- 8	+ 11000
+ 1	1 00001

2's complement of -8:

8_{10} = 01000 ⟶ 10111 (1's complement)

\+ 1

11000 (2's complement)

Binary multiplication

0 * 0 = 0

0 * 1 = 0

1 * 0 = 0

1 * 1 = 1

Note: remember decimal numbers multiplication.

Example-12: multiply the number$(111)_2$ to the number $(101)_2$

Solution:

```
      111
   *  101
   -------
      111
     0000
 +  11100
 ---------
   100011
```

the result$(100011)_2$

Example-13: Multiply the number $(1.111)_2$ to the number $(1.001)_2$.

Solution:

```
      1.111
  *   1.001
  ---------
       1111
      00000
     000000
+   1111000
  ---------
  10.000111
```

Binary division

Example-14: Divide the number$(11110)_2$ by the number $(101)_2$

```
        110
     ________
101 |  11110
        101 -
     ________
       0101
         101 –
     ________
         0000   the result is (110)2
```

Arithmetical operation of octal number

- In the process of addition, if the sum is less than or equal to 7, then it can be directly written as an octal digit.
- If the sum is greater than 7, then subtract 8 from the digit and carry 1 to the next digit position.

- Note that in this addition the largest octal digit is 7.

Arithmetical operation of octal number

Octal addition:

Example-15: Add the number $(372)_8$ to number $(245)_8$.

Solution:

$$
\begin{array}{r}
1 \\
372 \\
+\ 245 \\
\hline
637
\end{array}
\qquad (7+4)=11 \geq 8 \rightarrow 11-8=3
$$

the result is $(637)_8$

Arithmetical operation of octal number

$(136)_8 + (636)_8$

Solution:

```
  1        <---- carry

 136

 636
 ---
 774       <---- 6+6=12>8 in decimal, so in octal 6+6= 12-8=14 (4 and carry 1)
```

Therefore, sum = 774_8

Octal subtraction:

Example-16: subtract the number $(224)_8$ from the number $(735)_8$.

solution:

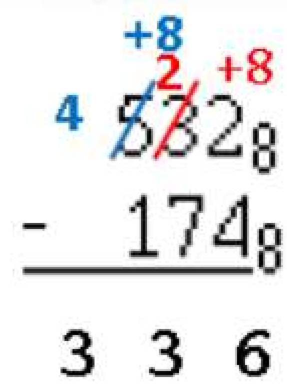
$$\begin{array}{r} 735 \\ -\ 224 \\ \hline 511 \end{array}$$

the result is $(511)_8$

Example-17

Subtract $532_8 - 174_8$

Solution:



Steps:

1. Since 2<4 then borrow 1 from 3 and add 8 to 2.
2. 2+8=10 in decimal so 10-4=6.
3. In the second column we have 2 after borrowing but 2<7, so we need to borrow 1 from 5 and add 8 to 2.
4. 2+8 = 10, 10-7=3
5. We have 4 after borrowing so we have 4-1=3.

3-Arithmetic operation on hexadecimal numbers

Hexadecimal addition:- follow the steps below:

- For any given hexadecimal digit think of their equivalent in decimal numbers representation.
- If the sum is (15 or less) in decimal it can be directly expressed as hexadecimal digits.
- If the sum is $\geq (16)_{10}$ subtract $(16)_{10}$ & carry "1" to the next position (digit)

Example-18: Find $(58)_{16} + (24)_{16}$

Solution:

```
  58
+ 24
----
  7C
```

Example-19: Add the number $(FF3)_{16}$ to number $(456)_{16}$.

```
   11
   FF3   (F+5)=20≥16→ 20 − 16 = 4
+  456   (F+4+1)=20≥16→ 20 − 16 = 4
------
```

$(1449)_{16}$ (Note that the carry=1)

Hexadecimal subtraction:

- can be directly found
- It can be found by converting to binary , then converting back to hexadecimal.
- using the 1's & 2's complement in hexadecimal.

Example-20: subtract the number$(CDF.26)_{16}$ From the number$(AC44.22)_{16}$.

Solution:

$$
\begin{array}{r}
AC44.22 \\
-\quad CDF.26 \\
\hline
\end{array}
$$

$(\ 9F64.FC\ )_{16}$ (note that borrow=0)

Using 1's &2's complement in hexadecimal numbers:

- 1's complement= FFF- No$(xxx)_{16}$
- 2's complement= 1's complement+1

Example-21: Find 2's complement of the hexadecimal number $(3A5)_{16}$.

Solution:

	FFF
	-3A5
1'S complement	C5A
	+1
2'S Complement	C5B

4-Arithmetical operation on decimal numbers

Decimal numbers complements:-

9’s complement: it is obtained by subtracting each digit from 9.

10’s complement: it is obtained by adding 1 to the 9’s complement value.

Example-22:

9’s complement of 546700 is 453299

10’s complement of 546700 is 453300

Note: complement are used in digital computers for simplifying the subtraction operation.

Rules:-

A-For M > N, the sum produce an end carry, which can be discarded and the left is the result 0f (M-N).

B- For M < N, The sum does not produce an end carry and to obtained the answer in a familiar form, take the 10’s complement of the sum and place a negative sign in front .

Example-23: Using 10's complement subtract:

$(72532)_{10} - (3250)_{10} = (\qquad)_{10}$

M > N

$$
\begin{array}{rr}
M = & 72532 \\
\text{10'S complement of } N & +\,96750 \\
\hline
 & 1\ 69282
\end{array}
$$

Discard the end carry 1 & the result is $(69282)_{10}$

Example-24:Using 10’s complement subtract:

$(3250)_{10} - (72532)_{10} = (\quad)_{10}$

M < N

M = 03250

10’S complement of N + 27468

30718

There is no carry , therefore the answer is – (10’s complement of the sum 30718)

The result= - 69282

Reference:
Digital Fundamentals, Thomas L. Floyd, chapter two, Eleventh Edition, 2015, Pearson

Thank you
with best wishes