Boolean algebra

Objectives:

1- Basic rules of Boolean algebra.

2- De Morgan's theorems.

3- Boolean expression for a logic circuit.

4- The sum of product (SOP) form.

5- The product of sum (POS) form.

6- Examples

1-Basic rules of Boolean algebra

1.1- Boolean Addition (OR gate)

A + 0 = A

A + 1 = 1

A + A = A

A + $\bar{A} = 1$

1.2- Boolean Multiplication (AND gate)

A . 0 = 0

A . 1 = A

A . A = A

A . $\bar{A}$ = 0

1.3-Commutative laws

A + B = B + A   A . B = B . A

1.4- Associative laws

A + (B + C) = (A + B) + C

A . (B . C)= (A . B) . C

1.5-Distribution law

A . (B + C) = AB+ AC   (Left distribution law)

(B + C) . A = BA + CA   (Right distribution law)

1.6- Double complement theorem

$\bar{\bar{A}} = A$ $\bar{\bar{\bar{A}}} = \bar{A}$

2- De Morgan's theorem

$$\overline{(A + B)} = \bar{A} \, . \, \bar{B}$$

$$\overline{(A . B)} = \bar{A} + \bar{B}$$

Other theorem can be derived from the basic laws above:

1- A+AB=A

2- $AB+A\overline{B}=A$

3- $(A+\overline{B}).B=AB$

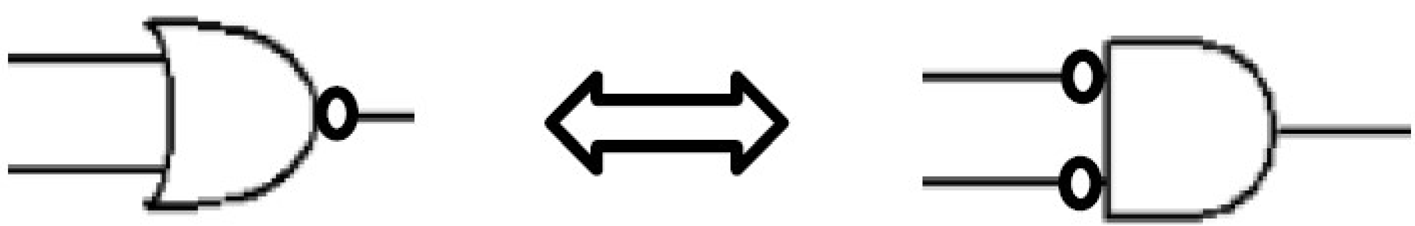
4- $(A+A).(A+\overline{B})=A$

5- (A+B).(A+C)=A+BC

6- A(A+B)=A

7- A+$\bar{A}$B=A+B

Example-1: prove $\overline{A + B}$ = $\bar{A}$. $\bar{B}$



X	Y	$\bar{A}$	$\overline{B}$	$\overline{(A+B)}$	$\bar{A}.\overline{B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

Example-2: Apply De Morgan's theorem to the expressions $\overline{XYZ}$ and $\overline{X+Y+Z}$

Solution:

1) $\overline{XYZ} = \bar{X} + \bar{Y} + \bar{Z}$

2) $\overline{X+Y+Z} = \bar{X}\ \bar{Y}\ \bar{Z}$

Home work: Apply De Morgan theorems to each of the following expressions.

1) $\overline{(A + B + C)D}$

2) $\overline{ABC + DEF}$

3) $\overline{\overline{(A + B)}} + \overline{\overline{C}}$

Home work: Apply De Morgan’s theorems to each of the following expressions.

(a) $\overline{(\overline{A + B}) + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

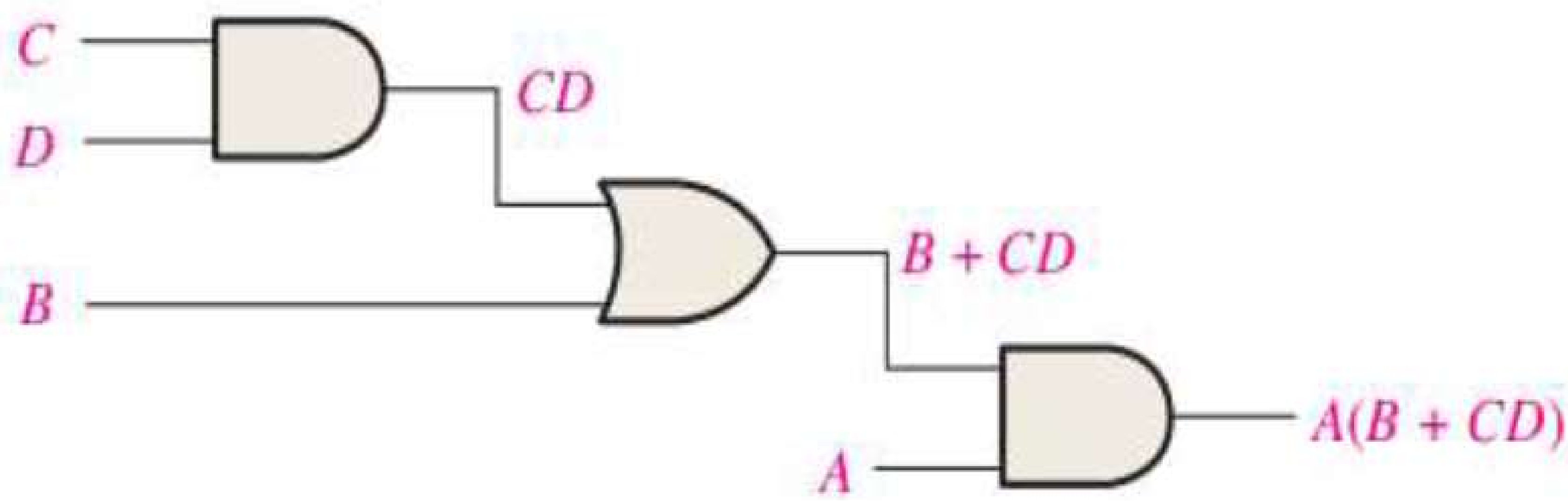
(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

3- Boolean expression for logic circuit:

Example-3:

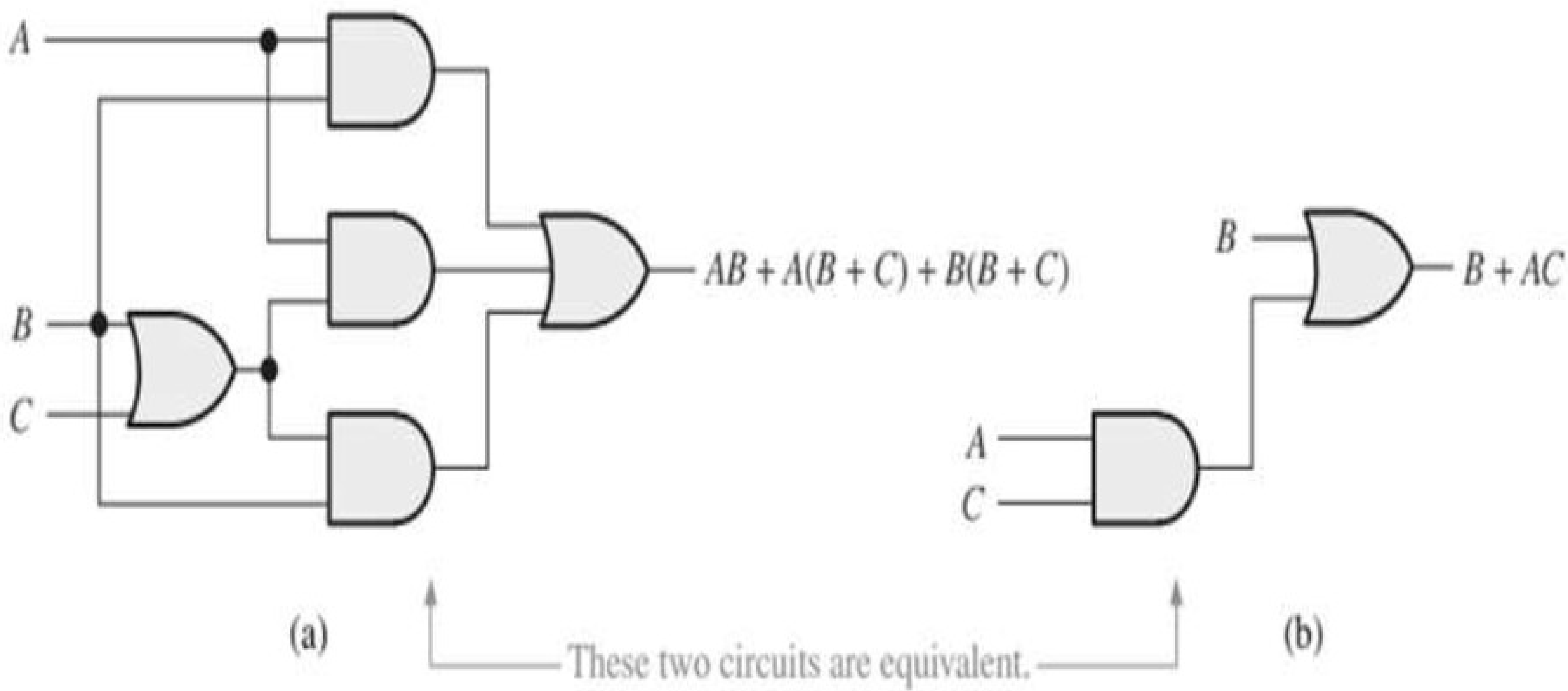
For the example circuit in Figure below, the Boolean expression is determined in the following three steps:

1. The expression for the left-most AND gate with inputs C and D is CD.
2. The output of the left-most AND gate is one of the inputs to the OR gate and B is the other input. Therefore, the expression for the OR gate is B + CD.
3. The output of the OR gate is one of the inputs to the right-most AND gate and A is the other input. Therefore, the expression for this AND gate is A(B + CD), which is the final output expression for the entire circuit.


C
D
CD
B
B + CD
A
A(B + CD)

Example-4: Using Boolean algebra techniques, simplify this expression and draw the logic circuit

$$F=(AB+A(B+C)+B(B+C)$$



Solution:

$$AB + A(B + C) + B(B + C)$$

$$= AB + AB + AC + BB + BC$$

$$= (AB + AB) + AC + (B + BC)$$

$$= AB + AC + B$$

$$= B + AC$$

Example-5: Simplify the following Boolean expression: $\overline{AB + AC} + \bar{A}\,\bar{B}C$

Solution:

$$\overline{(AB)}\;\overline{(AC)} + \bar{A}\;\bar{B}C$$

$$=(\bar{A} + \bar{B})(\bar{A} + \bar{C}) + \bar{A}\;\bar{B}\,C$$

$$=\bar{A}\;\bar{A} + \bar{A}\;\bar{C} + \bar{A}\;\bar{B} + \bar{B}\,\bar{C} + \bar{A}\;\bar{B}\,C$$

$$=\bar{A} + \bar{A}\;\bar{C} + \bar{A}\;\bar{B} + \bar{B}\,\bar{C}$$

$$=\bar{A} + \bar{B}\;\bar{C}$$

Home work: Draw the logic diagrams for the following Boolean expressions

1) $Y=\bar{A}\bar{B}+B(A+C)$
2) $Y=BC+A\bar{C}$
3) $Y=A+CD$
4) $Y=(A+B)(\bar{C}+D)$

4-The sum 0f products (SOP) Form

A product term was defined as term consisting of product of variable or their complements. When two or more product terms are summed by Boolean addition, the resulting expression is a sum of product (SOP). Some example are:

$$AB + ABC$$

$$ABC + CDE + \bar{B}C\bar{D}$$

$$\bar{A}B + \bar{A}B\bar{C} + AC$$

5-The product of sums (POS) Form

A sum term was defined as term consisting of sum of variables or their complements. When two or more sum terms are multiplied, the resulting expression is a product of sums (POS). Some example are:

$$(\bar{A} + B)(A + \bar{B} + C)$$

$$(\bar{A} + \bar{B} + \bar{C})(C + \bar{D} + E)(\bar{B} + C + D)$$

$$(A + B)(A + \bar{B} + C)(\bar{A} + C)$$

Consider the truth table (T.T) for three variables given below using S0P and POS

X	Y	Z	SOP (Minterms)	Designation	POS (Maxterms)	Designation
0	0	0	$\bar{X}\bar{Y}\bar{Z}$	m0	$X+Y+Z$	M0
0	0	1	$\bar{X}\bar{Y}Z$	m1	$X+Y+\bar{Z}$	M1
0	1	0	$\bar{X}Y\bar{Z}$	m2	$X+\bar{Y}+Z$	M2
0	1	1	$\bar{X}YZ$	m3	$X+\bar{Y}+\bar{Z}$	M3
1	0	0	$X\bar{Y}\bar{Z}$	m4	$\bar{X}+Y+Z$	M4
1	0	1	$X\bar{Y}Z$	m5	$\bar{X}+Y+\bar{Z}$	M5
1	1	0	$XY\bar{Z}$	m6	$\bar{X}+\bar{Y}+Z$	M6
1	1	1	XYZ	m7	$\bar{X}+\bar{Y}+\bar{Z}$	M7

Example-6: Drive Boolean expression (B.E) using SOP and POS methods for 3-inputs & the output will be high (1) when the binary input values equal (1,4,7).

Solution:

In SOP; $F=\sum m\ (1,4,7)=m1+m4+m7=\bar{X}\bar{Y}Z + X\bar{Y}\bar{Z} + XYZ$

In POS; $F=\pi M\ (0,2,3,5,6)$

$=(X+Y+Z).(X+\bar{Y}+Z).(X+\bar{Y}+\bar{Z}).(\bar{X}+Y+\bar{Z}).(\bar{X}+\bar{Y}+Z)$

Example-7: From the truth table below, determine the standard SOP expression and the equivalent standard POS expression.

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1

Solution: There are four 1s in the output:

$011 = \bar{X}YZ$

$100 = X\bar{Y}\bar{Z}$

$110 = XY\bar{Z}$

$111 = XYZ$

$$F = \bar{X}YZ + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$$

(For the SOP expression, the output is 1)

For the POS expression, the output is 0:

$000 = X + Y + Z$

$001 = X + Y + \bar{Z}$

$010 = X + \bar{Y} + Z$

$101 = \bar{X} + Y + \bar{Z}$

The resulting standard POS expression for the output F is

$F = (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})$

Example-8: Design a logic circuit with 3-inputs and the output will be high (1) when the binary input values less than 3 using SOP & POS methods.

Solution:

X	Y	Z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

SOP: $F=\bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z+\bar{X}Y\bar{Z}$

POS: $F = (X + \bar{Y} + \bar{Z})(\bar{X} + Y + Z)(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + Z))(\bar{X} + \bar{Y} + \bar{Z})$

Example- 9: Design a logic circuit with 3-inputs & the output will be high (1) when a majority of inputs are high (1) using SOP & POS methods;

Solution:

SOP: $F=\bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$

POS: $F= (X + Y + Z)(X + Y + \bar{Z})(X + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})$

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

1) References

1) Digital Fundamentals, Thomas L. Floyd, Eleventh Edition, 2015, Pearson.
2) Digital design, M. Morris Mano and Michael D. Ciletti, Fifth Edition, 2013, Pearson.

Thank you
With best wishes