Karnaugh map(k-map)

Objectives

At the end of the lecture student may learn:

1- Draw the corresponding Karnaugh map.

2- rules of simplification of Boolean expressions using Karnaugh map.

3- Don’t care conditions.

4- Examples

1- Draw the K-map

- There must be 2^n squares in the K-map for n input variables.
- Plot given function on the map.
- Combine those terms of which there is on possibility first, and so on for all terms.
- The combinations will be at least 2 as high as 4,8,16 is better.
- The combining of 2-minterms (pairs) will eliminate one variable.

- The combining of 4-minterms (quads) will eliminate two variables.
- The combining of 8-minterms (octal) will eliminate three variables.
- Each cell in a group must be adjacent to one or more cells in the same group, but all cells in the group do not have to be adjacent to each other.

❖ Two variable K-map: $2^n = 2^2 = 4$-squares

X	Y	Minterms
0	0	m0
0	1	m1
1	0	m2
1	1	m3

X \ Y	0	1
0	m0	m1
1	m2	m3

- Three variable K-map: $2^n=2^3=8$-squares

X	Y	Z	Minterms
0	0	0	m0
0	0	1	m1
0	1	0	m2
0	1	1	m3
1	0	0	m4
1	0	1	m5
1	1	0	m6
1	1	1	m7

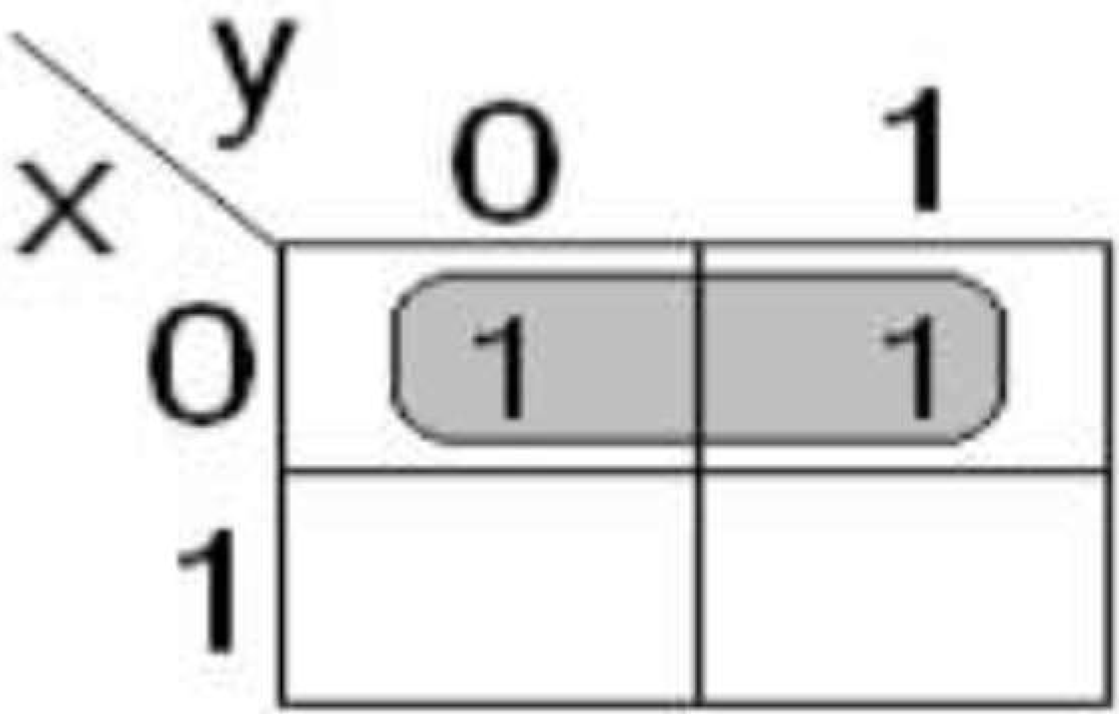
X \ YZ	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

- Four variable K-map: $2^n=2^4=16$-squares

WX \ YZ	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m10

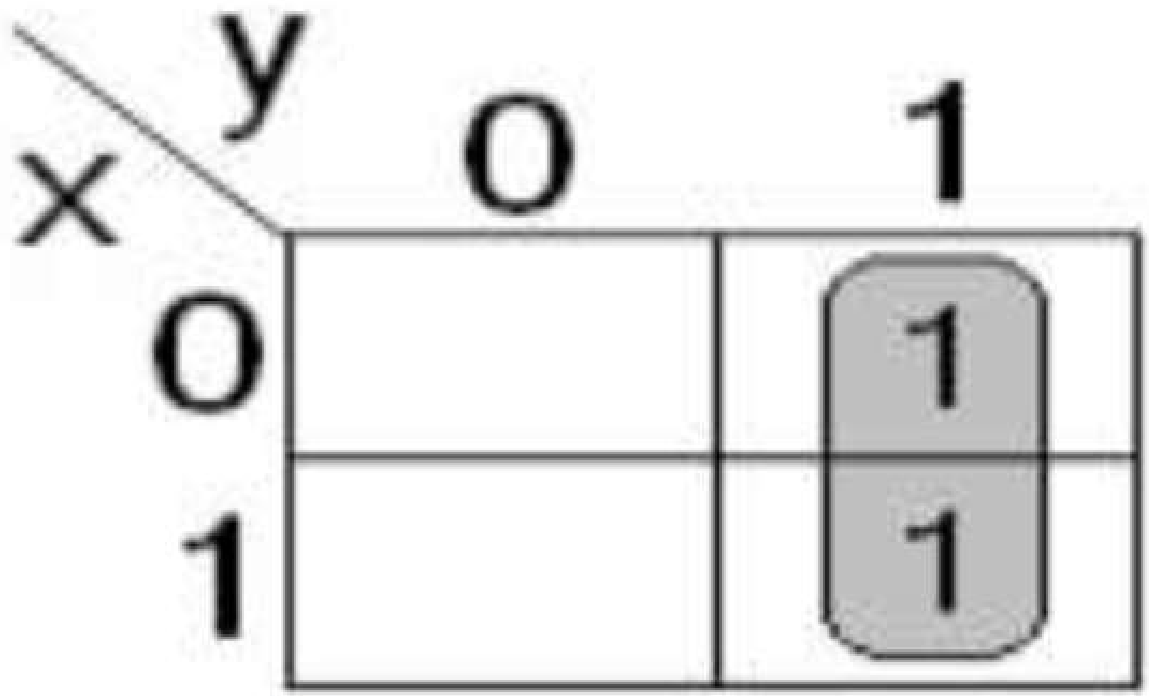
Example-1: Simplify the following Boolean functions.

$$1\text{-}F(X,Y) = X'Y + X'Y'$$



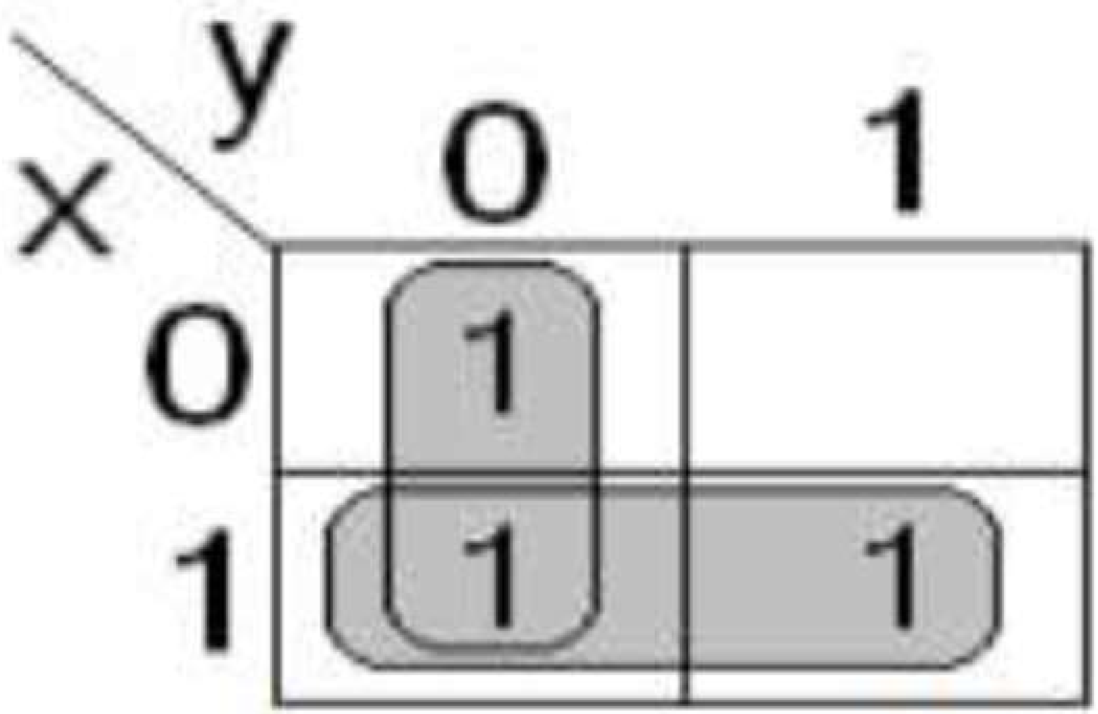
$$F(x,y) = x'$$

2- $F(X,Y) = XY + X'Y$



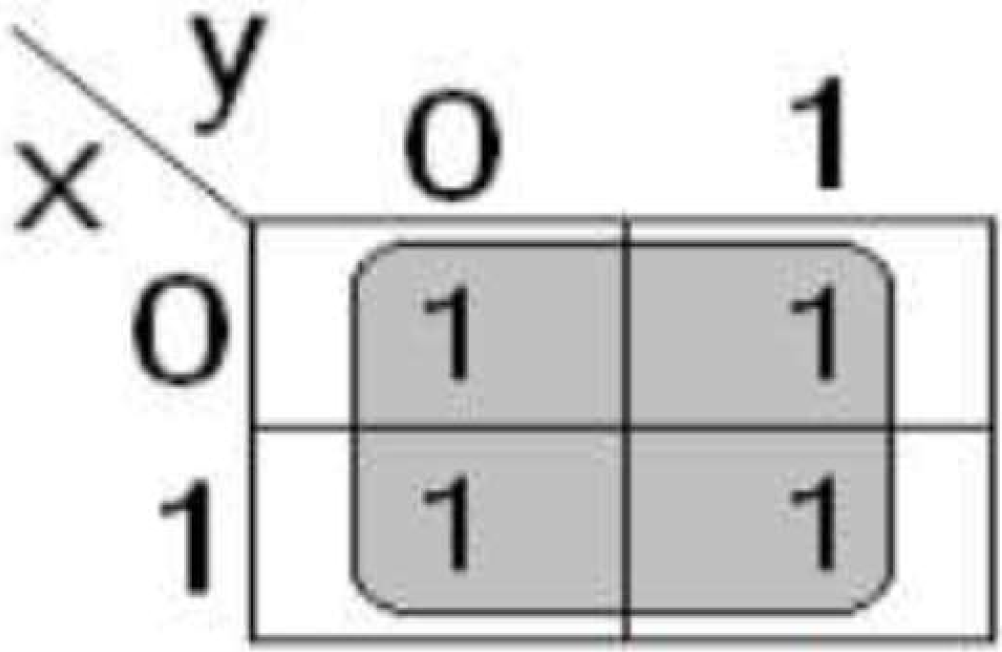
$F(x,y) = y$

$$3\text{-}F(X,Y) = X'Y' + XY' + XY$$



$$F(x,y) = x + y'$$

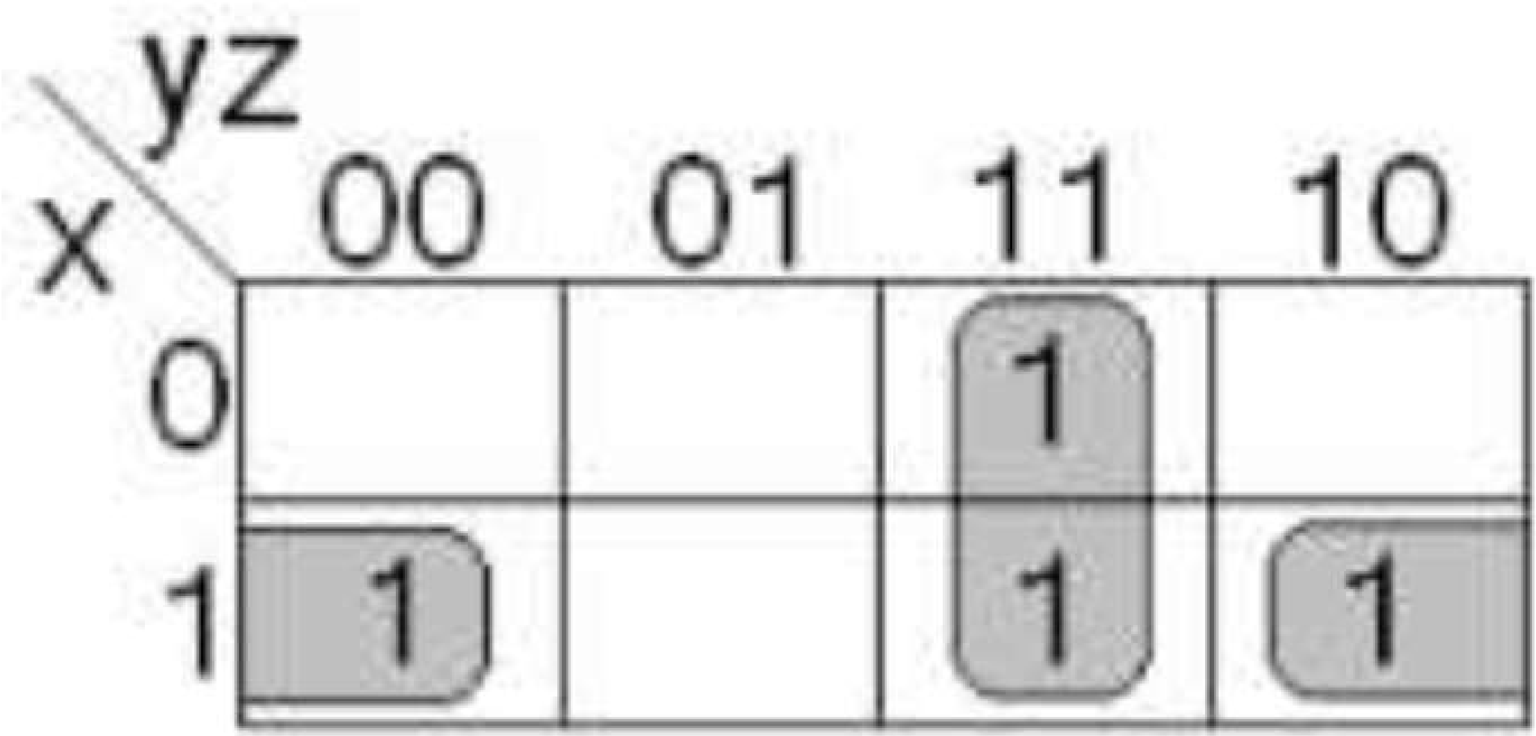
4- $F(X,Y) = XY + X'Y + XY' + X'Y'$



$F(x,y) = 1$

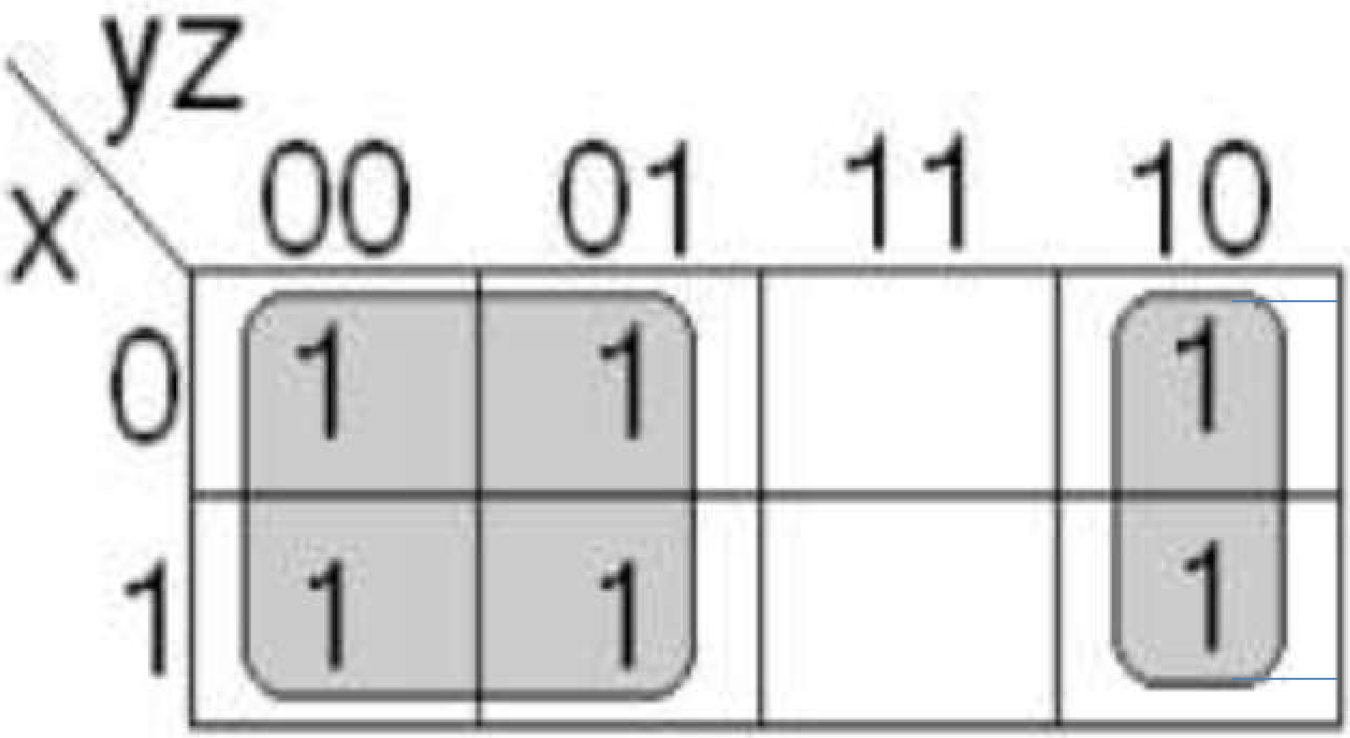
Example-2: Simplify the following Boolean functions.

1- $F(x,y,z) = (3,4,6,7)$



$F(x,y,z) = xz' + yz$

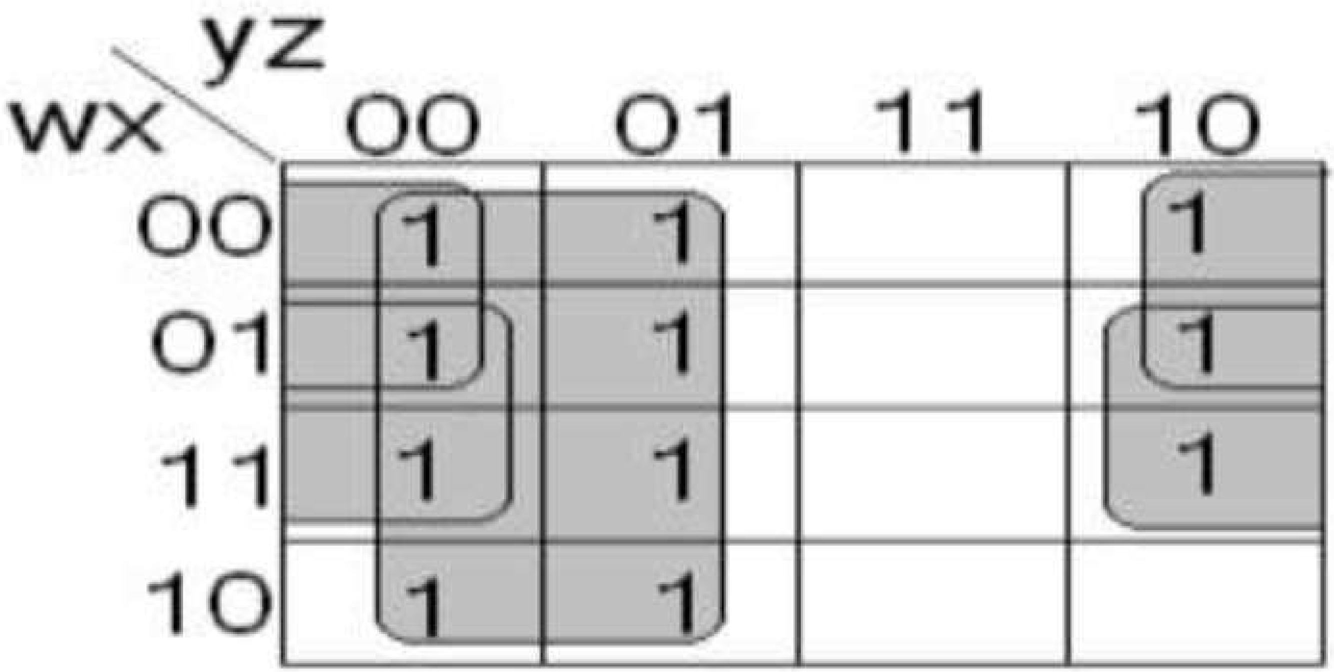
2- $F(x,y,z) = (0,1,2,4,5,6)$



$$F(x,y,z) = y' + z'$$

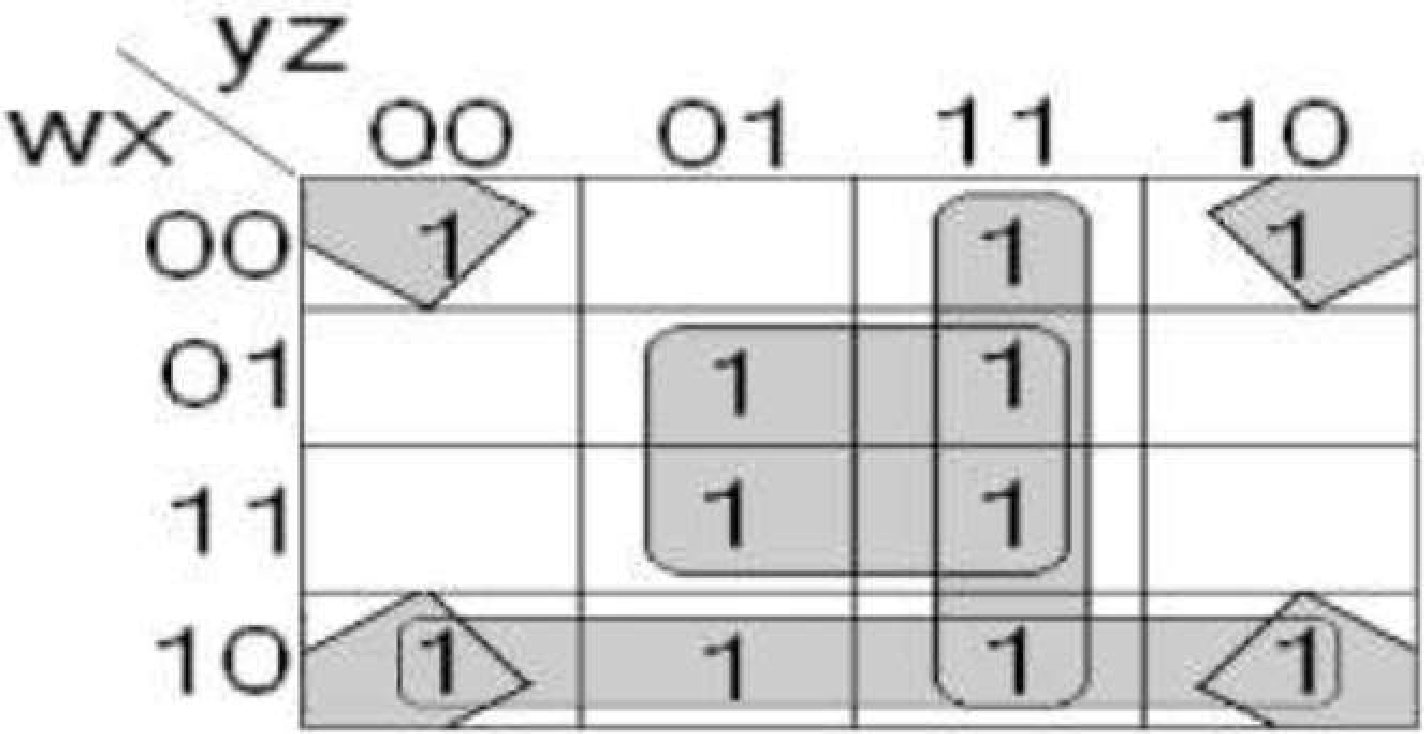
Example-3: Simplify the following Boolean functions

1- $F(w,x,y,z) = \sum m\ (0,1,2,4,5,6,8,9,12,13,14)$



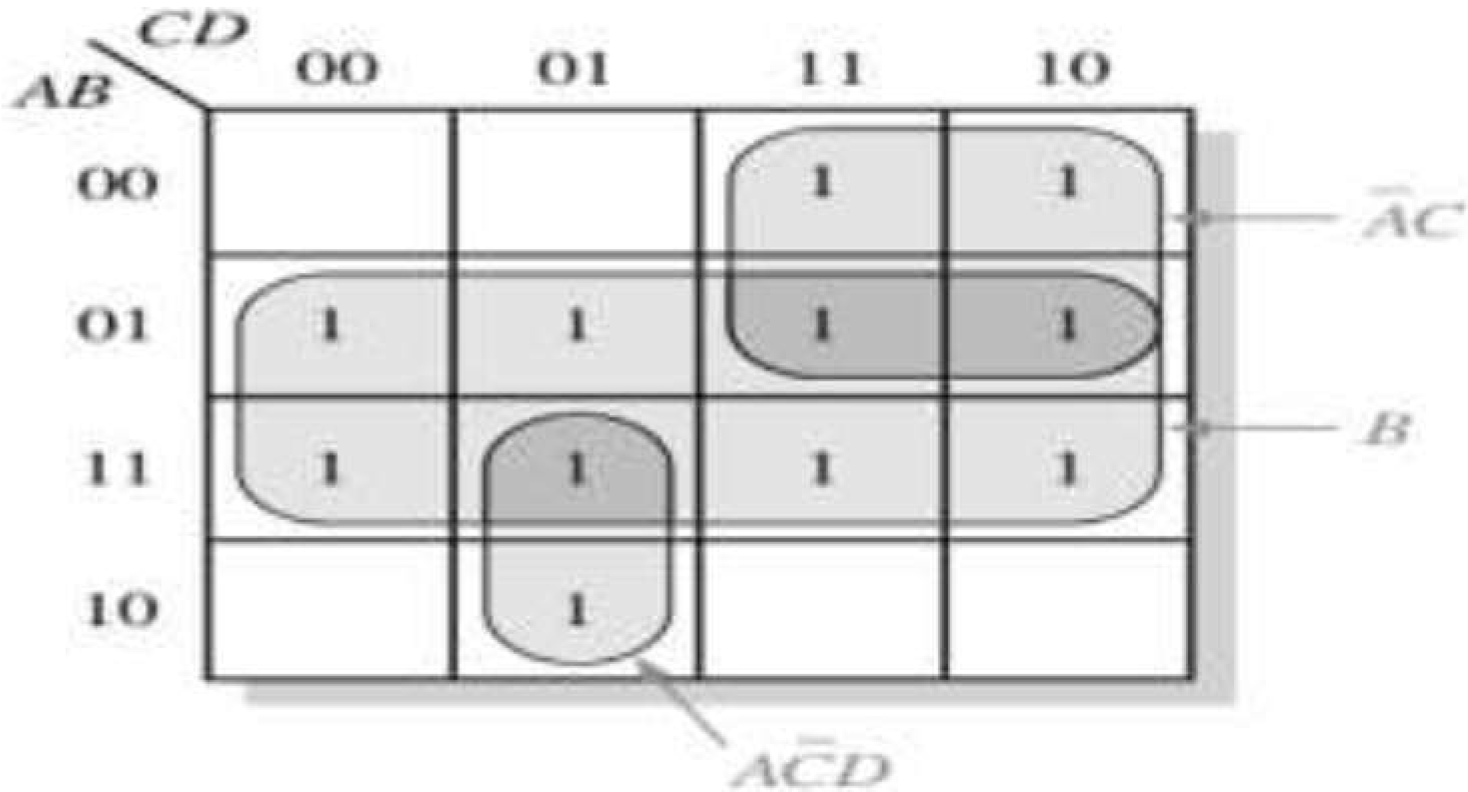
$F(w,x,y,z) = y' + w'z' + xz'$

2-$F(w,x,y,z) = \sum m\ (0,2,3,5,7,8,9,10,11,13,15)$

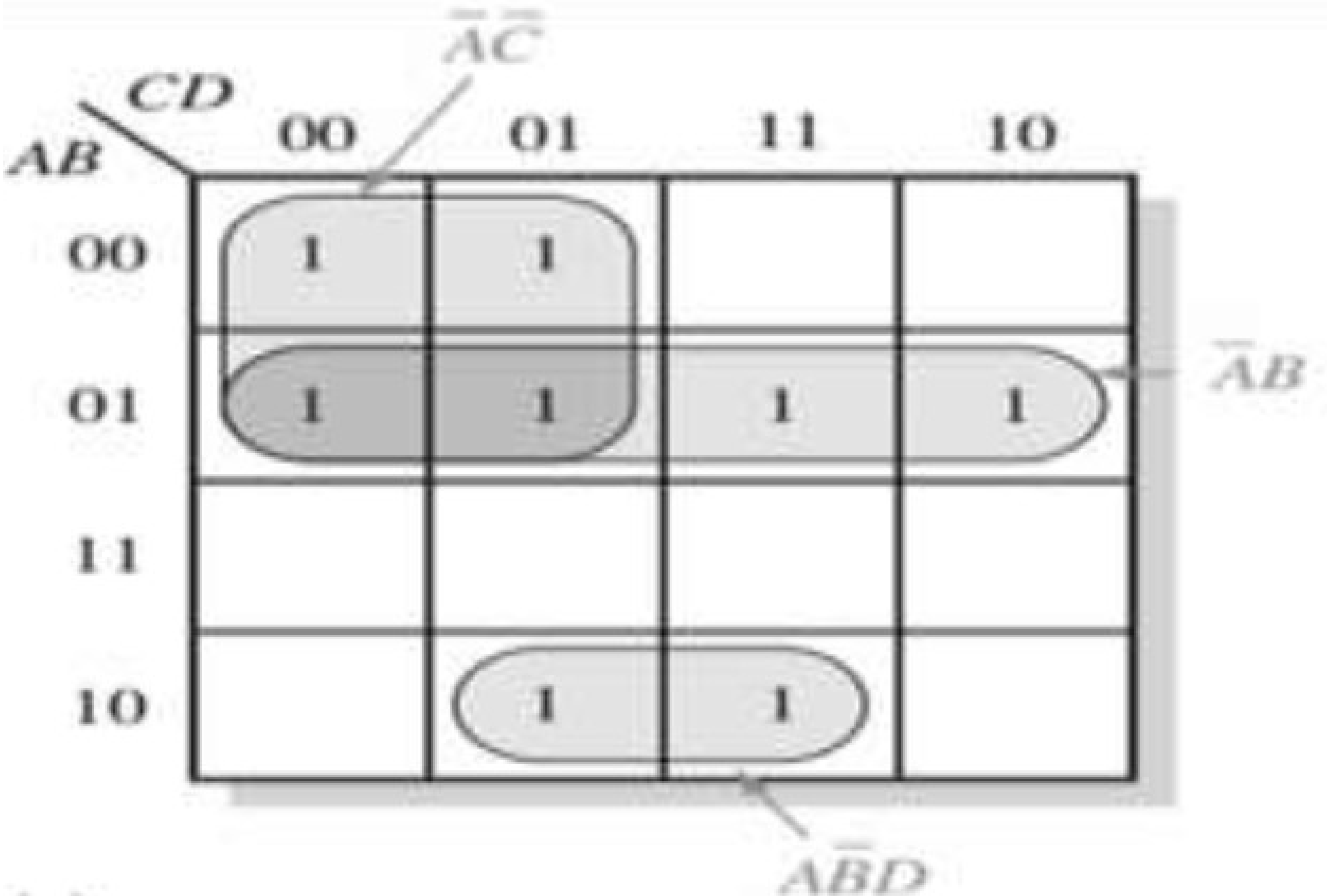


$F(w,x,y,z) = wx' + yz + xz + x'z'$

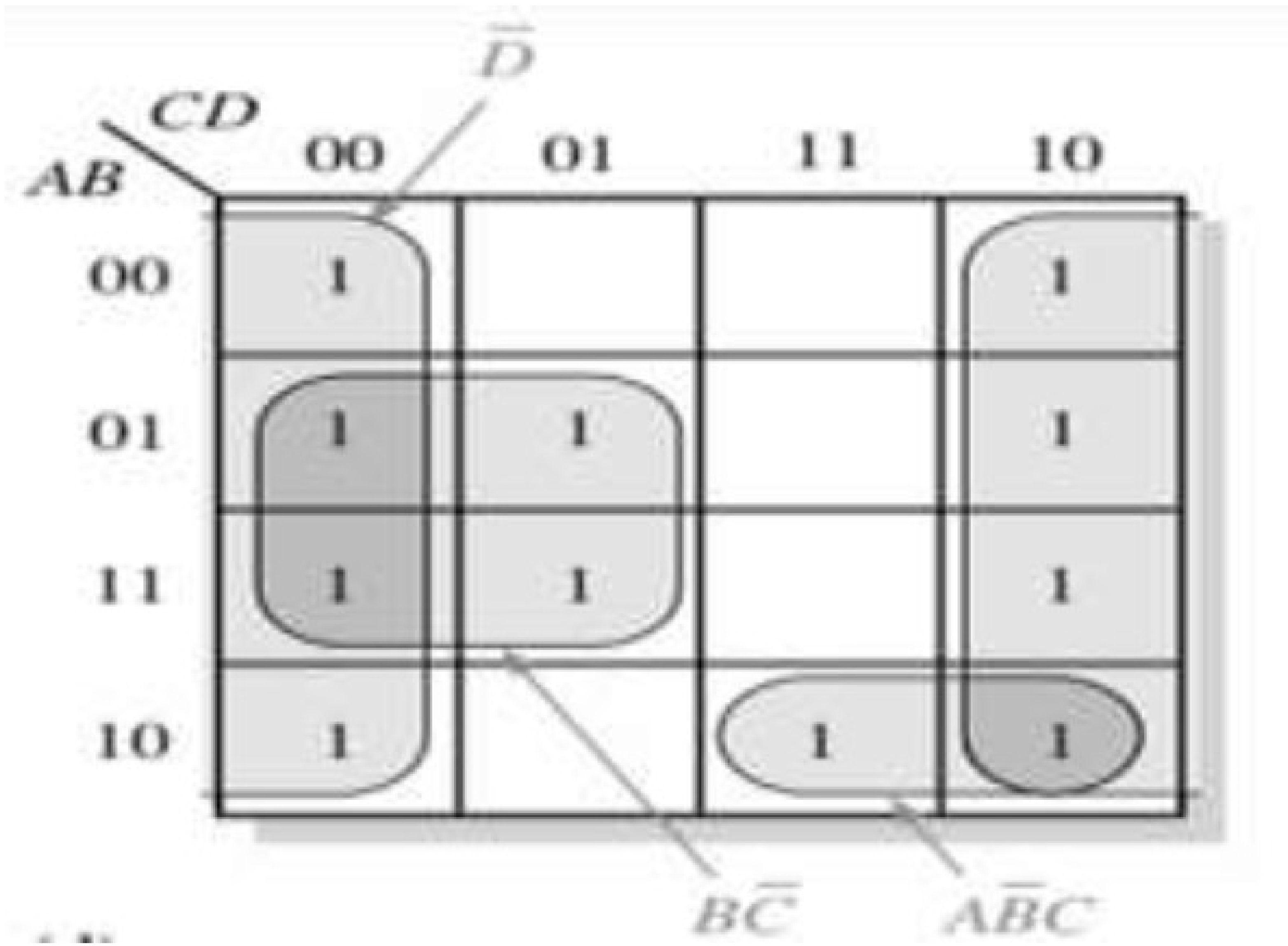
Example-4: Determine the product terms for K-map and write the resulting minimum SOP expression.



$$F=B + \bar{A}C + A\bar{C}D$$



$$F = \bar{A}B + \bar{A}\bar{C} + A\bar{B}D$$



$$F = \bar{D} + A\bar{B}C + B\bar{C}$$

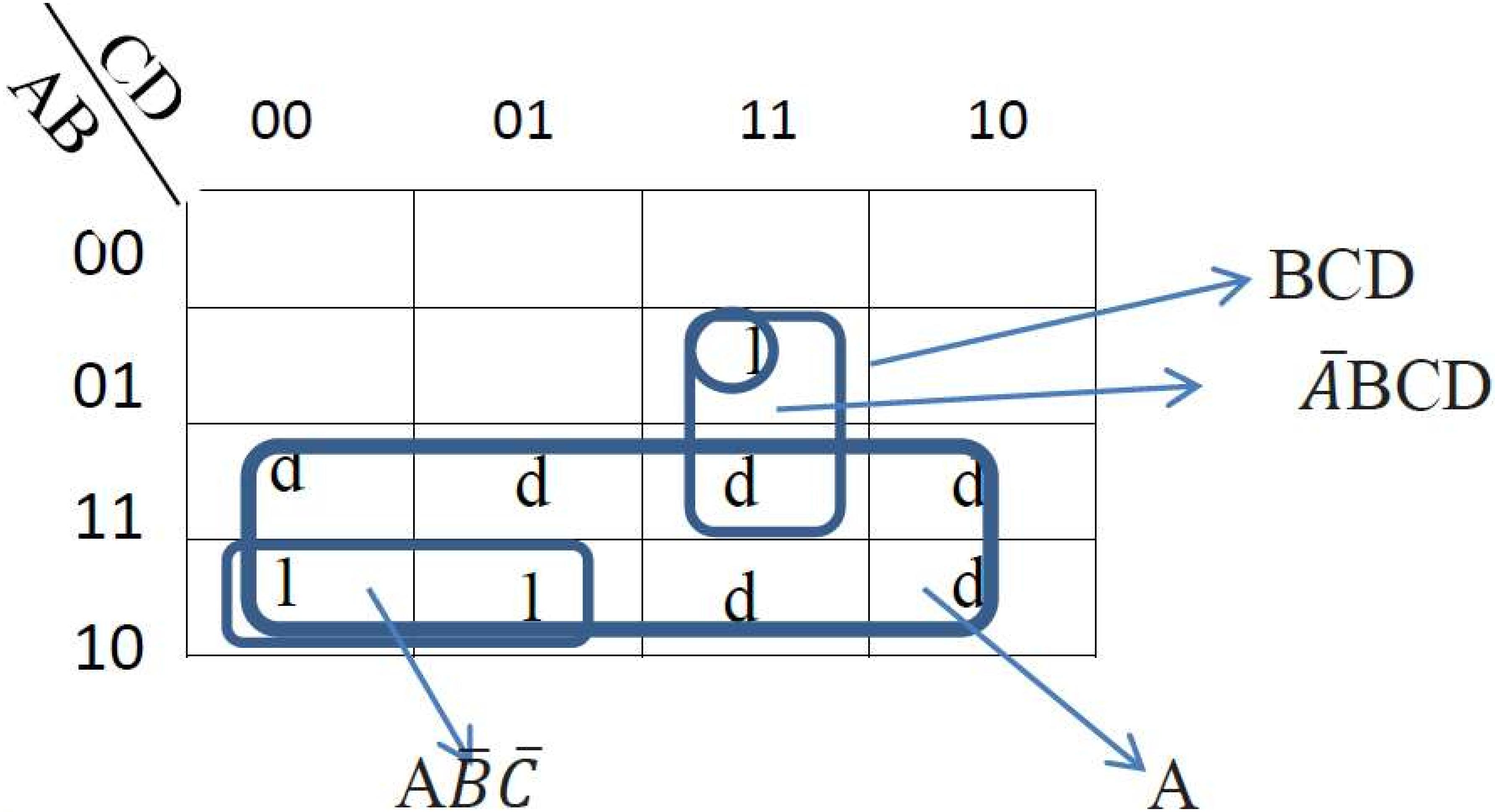
2- Don’t Care condition

Sometimes a situation arises in which some input variable combinations are not allowed. For example, recall that in the BCD code, there are six invalid combinations (1010,1011,1100,1101and 1111). They can be treated as “ don’t care” terms with respect to their effect on the output. That is , for these “don’t care” terms either a 1 or a 0 may be assigned to the output. The “don’t care” terms can be used to advantage on the K- map.

Example-: The truth table below describes a logic function that has a 1 output only when BCD code for 7,8,9 is present on the inputs.

- ✓ If the "don't cares" are used as 1s. The resulting expression for the function is [A+BCD]
- ✓ If the "don't cares" are not used as 1s. The resulting expression is [$A\bar{B}\bar{C} + \bar{A}BCD$]

Input				Output
A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	d
1	0	1	1	d
1	1	0	0	d
1	1	0	1	d
1	1	1	0	d
1	1	1	1	d



- Without "don't cares"F=[$A\bar{B}\bar{C} + \bar{A}BCD$];
- With "don't cares" F= [A + BCD],

Example: Simplify using K-map:-

$F(A,B,C) = \sum m\ (0,4) + (d=1,2,5,7)$

Solution: $F = \bar{B}$

A \ BC	00	01	11	10
0	1	d		d
1	1	d	D	

Example: Simplify using K-map:-

$F(A,B,C,D) = \sum m\ (1,4,6,11,12) + (d = 0,2,3,9,14)$

AB \ CD	00	01	11	10
00	d	1	d	d
01	1			1
11	1			d
10		d	1	

$\bar{B}D$

$B\bar{D}$

$$F = B\bar{D} + \bar{B}D = B \oplus D$$

Example: Simplify the following Boolean function by using K-map

1-F(x,y,z) = (0,1,2,4,6), d(x,y,z) = (3,7, 8)

x \ yz	00	01	11	10
0	1	1	d	1
1	1	d	d	1

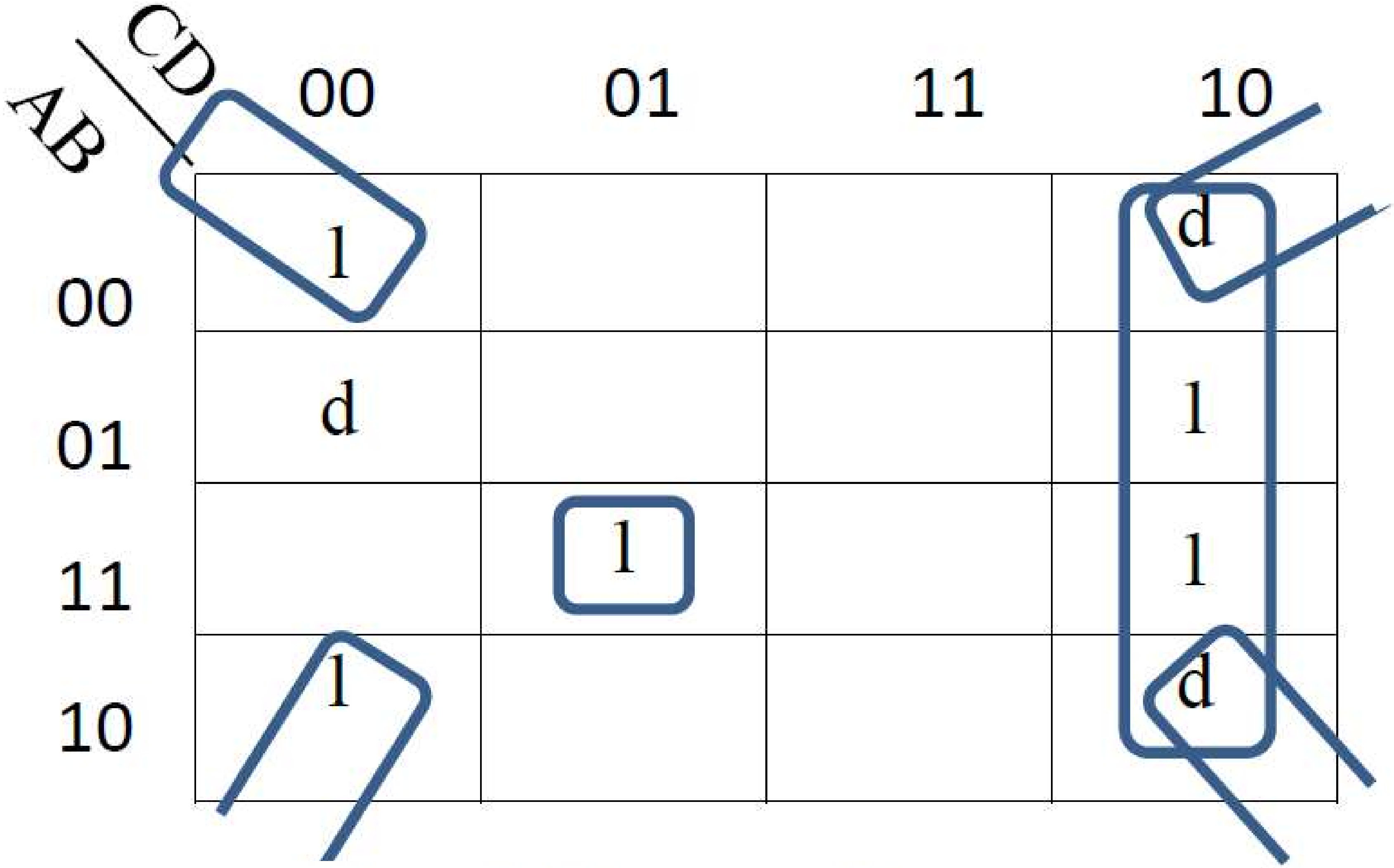
F=1

2-F(A,B,C,D) = (1,3,5,7,9,15), d(A,B,C,D) = (4,6,12,13)

AB \ CD	00	01	11	10
00		1	1	
01	d	1	1	d
11	d	d	1	
10		1		

$$F = \bar{C}D + \bar{A}D + BD$$

3-F(A,B,C,D) = (0,6,8,13,14), d(A,B,C,D) = (2,4,10)



F= $C\overline{D} + \overline{B}\,\overline{D}$ +AB$\bar{C}$D

References

1) Digital Fundamentals, Thomas L. Floyd, Eleventh Edition, 2015, Pearson
2) Digital Design, M. Morris Mano and Michael D. Ciletti, Fifth Edition, 2013, Pearson

Thank you

With best wishes